Principles of Computer Science II

Sorting Algorithms

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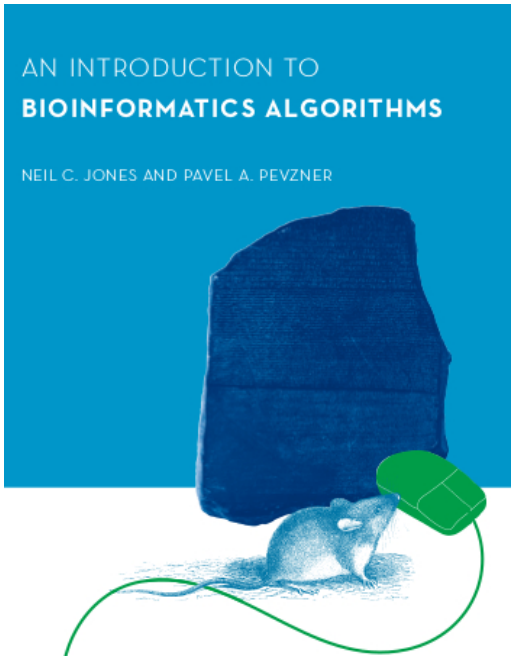
Lecture 3

Sorting Problem

Jones, Pevzner: An Introduction to Bioinformatics Algorithms. MIT Press, 2004

Section 2.6 - Sorting Problem



Sorting Problem:
Sort a list of integers.

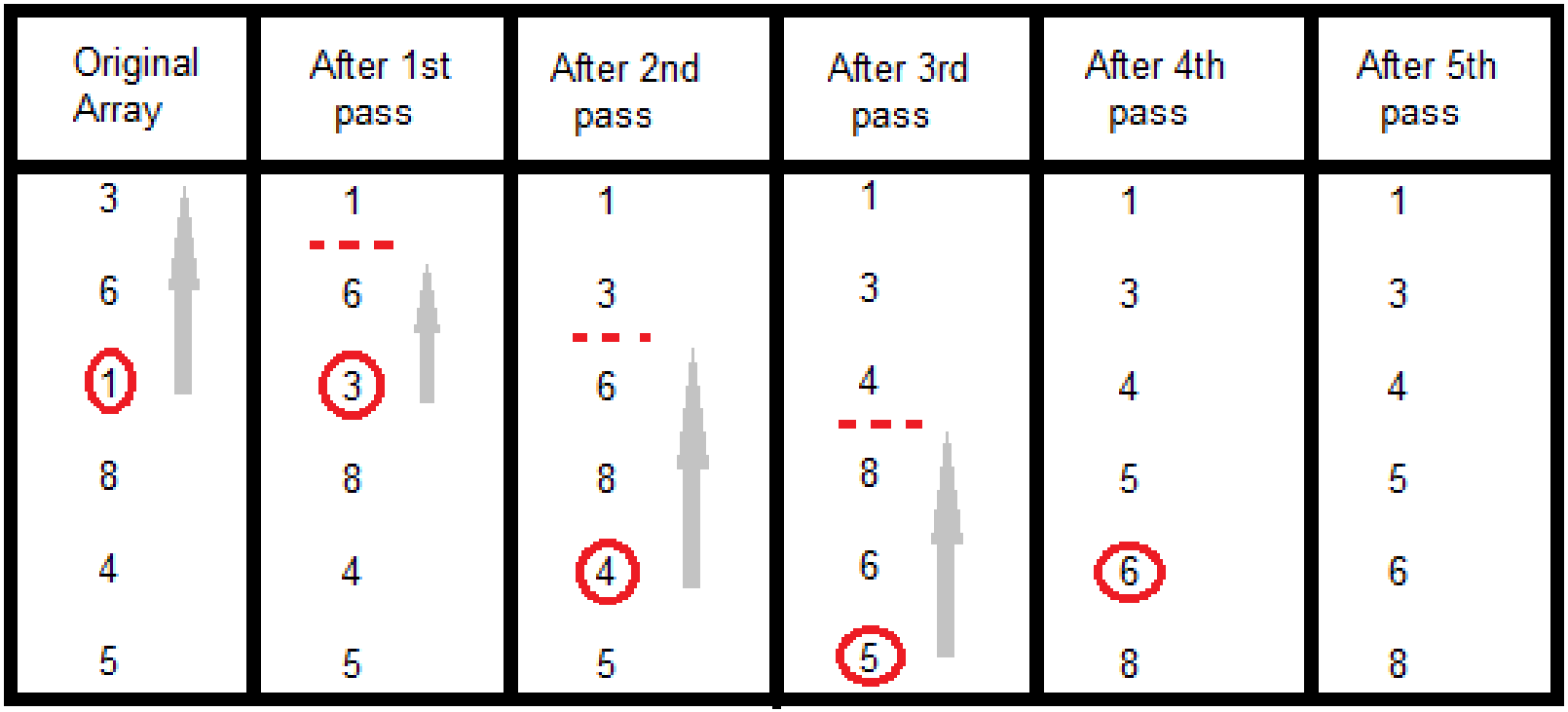
Input: A list of n distinct integers $\mathbf{a} = (a_1, a_2, \ldots, a_n)$.

Output: Sorted list of integers, that is, a reordering $\mathbf{b} = (b_1, b_2, \ldots, b_n)$ of integers from $\mathbf{a}$ such that $b_1 < b_2 < \cdots < b_n$.

Selection Sort Algorithm

This algorithm first finds the smallest element in the array and exchanges it with the element in the first position, then find the second smallest element and exchange it with the element in the second position, and continues in this way until the entire array is sorted.

Selection Sort: Example



Selection Sort Code

```
a = [5, 1, 6, 2, 4, 3]
for i in range(0, len(a)):
    min = i
    for j in range(i + 1, len(a) - 1):
        if a[j] < a[min]:
            min = j

    temp = a[j]
    a[j] = a[min]
    a[min] = temp
```

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- How much memory is needed ?
  - 2 additional slot (min and temp) - constant!

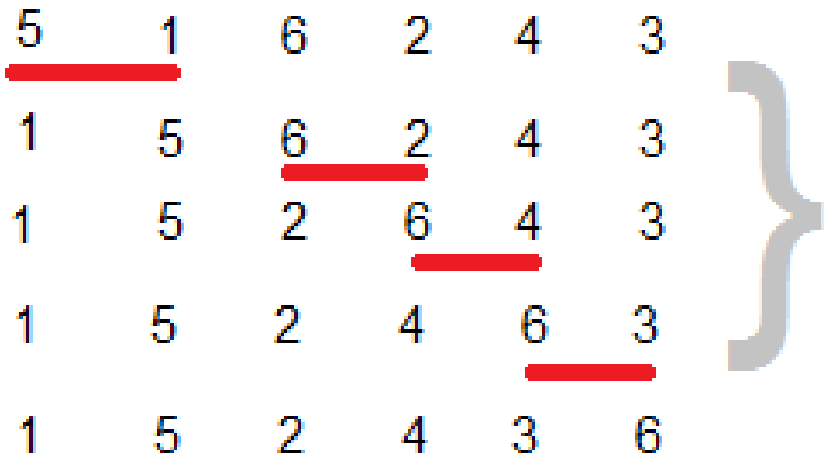
Bubble Sort Algorithm

Bubble Sort is an algorithm which is used to sort N elements that are given in a memory. Bubble Sort compares all the element one by one and sort them based on their values.

- It is called Bubble sort, because with each iteration the largest element in the list bubbles up towards the last place, just like a water bubble rises up to the water surface.
- Sorting takes place by stepping through all the data items one-by-one in pairs and comparing adjacent data items and swapping each pair that is out of order.

Bubble Sorting: Example

Lets take this Array.



Here we can see the Array after the first iteration.

Similarly, after other consecutive iterations, this array will get sorted.

Bubble Sort Code

```
a = [5, 1, 6, 2, 4, 3]
for i in range(0, len(a)):
    for j in range(0, len(a) - i - 1):
        if a[j] > a[j+1]:
            temp = a[j]
            a[j] = a[j+1]
            a[j+1] = temp
```

- The above algorithm is not efficient because as per the above logic, the for-loop will keep executing for six iterations even if the list gets sorted after the second iteration.

Bubble Sort Code: Version 2

- We can insert a flag and can keep checking whether swapping of elements is taking place or not in the following iteration.
- If no swapping is taking place, it means the list is sorted and we can jump out of the for loop, instead executing all the iterations.

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for i in range(0, len(a)):
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  - N comparisons are required
- How much memory is needed ?
  - 1 additional slot (naive solution)
  - 2 additional slot (temp + flag in the optimized solution)
  - ...however, constant!

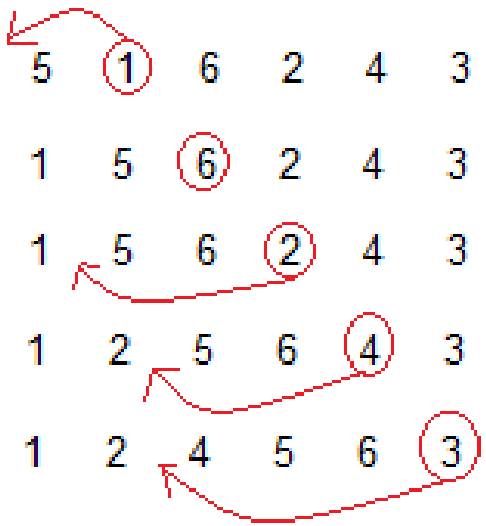
Insert Sort Algorithm

A simple Sorting algorithm which sorts the list by shifting elements one by one.

- It has one of the simplest implementation
- It is efficient for smaller data sets, but very inefficient for larger lists.
- Insertion Sort is adaptive, that means it reduces its total number of steps if given a partially sorted list, hence it increases its efficiency.
- It is better than Selection Sort and Bubble Sort algorithms.
- Like Bubble Sorting, insertion sort also requires a single additional memory space.

Insertion Sort: Example

Lets take this Array.



(Always we start with the second element as key.)

As we can see here, in insertion sort, we pick up a key, and compares it with elemnts ahead of it, and puts the key in the right place

5 has nothing before it.

1 is compared to 5 and is inserted before 5.

6 is greater than 5 and 1.

2 is smaller than 6 and 5, but greater than 1, so its is inserted after 1.

And this goes on...

Insertion Sort Code

```
a = [5, 1, 6, 2, 4, 3]
for i in range(1, len(a)):
    key = a[i]
    j = i - 1
    while j >= 0 and key < a[j]:
        a[j+1] = a[j]
        j -= 1
    a[j+1] = key
```

- key: we put each element of the list, in each pass, starting from the second element: a[1].
- using the while loop, we iterate, until j becomes equal to zero or we find an element which is greater than key, and then we insert the key at that position.

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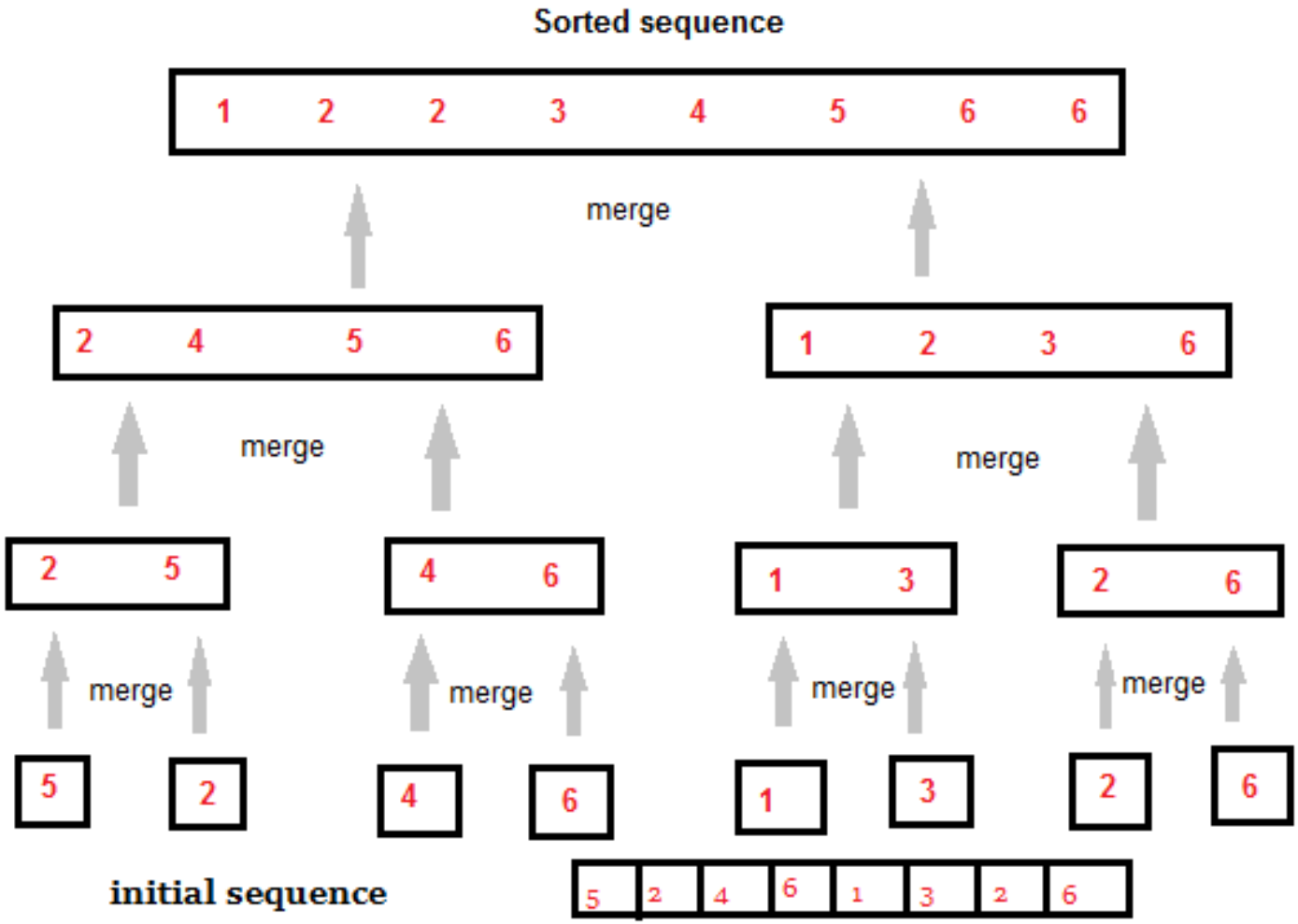
- How much memory is needed ?
  - 2 additional slot (key, j)
  - constant!

Merge Sort Algorithm

In Merge Sort the unsorted list is divided into N sublists, each having one element, because a list consisting of one element is always sorted. Then, it repeatedly merges these sublists, to produce new sorted sublists, and in the end, only one sorted list is produced.

- Divide and Conquer algorithm
- Performance always same for Worst, Average, Best case

Merge Sort: Example



Merge Sort Code

```
a = [25, 52, 37, 63, 14, 17, 8, 6]

def mergesort(list):
    if len(list) == 1:
        return list

    left = list[0: len(list) // 2]
    right = list[len(list) // 2:]

    left = mergesort(left)
    right = mergesort(right)

    return merge(left, right)
```

Merge Sort Code

```
def merge(left, right):
    result = []
    while len(left) > 0 and len(right) > 0:
        if left[0] <= right[0]:
            result.append(left.pop(0))
        else:
            result.append(right.pop(0))

    while len(left) > 0:
        result.append(left.pop(0))

    while len(right) > 0:
        result.append(right.pop(0))

    return result

print("Before: ", a)
r = mergesort(a)
print("After: ", r)
```

How good is Merge Sort?

- How many comparisons are required until the list is sorted?
  - 1^{st} loop: two lists $\frac{n}{2}$ each
  - 2^{nd} loop: four lists $\frac{n}{4}$ each
  - ...
  - $\log n$ steps
  - For each partition we do n comparisons
  - In total $n \log n$ comparisons
- How much memory is needed ?
  - 1 additional slot (result, larger).

Quick Sort Algorithm

Quick sort is very fast and requires very less additional space. It is based on the rule of Divide and Conquer. This algorithm divides the list into three main parts :

- Elements less than the Pivot element
- Pivot element(Central element)
- Elements greater than the pivot element

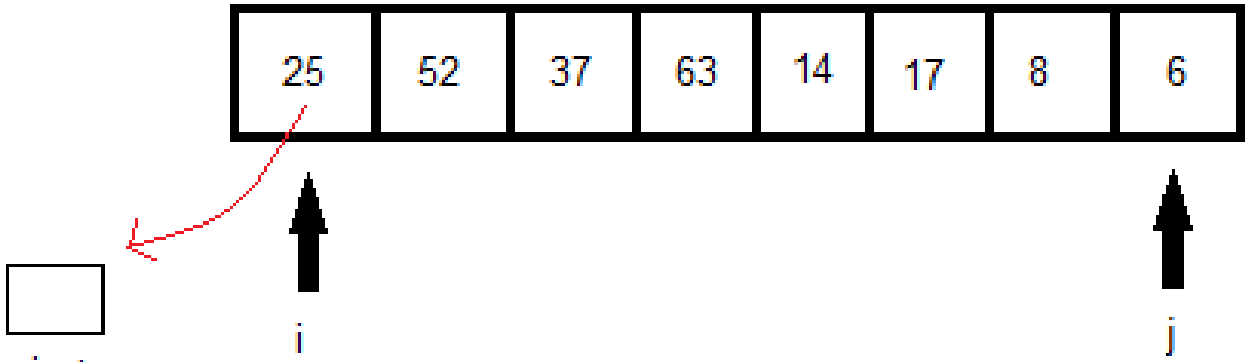
- Sorts any list very quickly
- Performance depends on the selection of the Pivot element

Quick Sort: Example

List: 25 52 37 63 14 17 8 6

- We pick 25 as the pivot.
- All the elements smaller to it on its left,
- All the elements larger than to its right.
- After the first pass the list looks like:
  6 8 17 14 25 63 37 52
- Now we sort two separate lists:
  6 8 17 14 and 63 37 52
- We apply the same logic, and we keep doing this until the complete list is sorted.

Quick Sort: Example



if both sides we find the element not satisfying their respective conditions, we swap them. And keep repeating this.

DIVIDE AND CONQUER - QUICK SORT

Quick Sort Code

```
a = [25, 52, 37, 63, 14, 17, 8, 6]

def partition(list, p, r):
    pivot = list[p]
    i = p
    j = r
    while(1):
        while(list[i] < pivot and list[i] != pivot):
            i += 1

        while(list[j] > pivot and list[j] != pivot):
            j -= 1

        if(i < j):
            temp = list[i]
            list[i] = list[j]
            list[j] = temp
        else:
            return j
```

Quick Sort Code

```
def quicksort(list, p, r):
    if (p < r):
        q = partition(list, p, r)
        quicksort(list, p, q);
        quicksort(list, q + 1, r);

print("Before: ", a)
quicksort(a, 0, len(a) - 1)
print("After: ", a)
```

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- How much memory is needed ?
  - 2 small additional slots.

Algorithm Design Techniques

Computer scientists have discovered that many algorithms share similar ideas, even though they solve very different problems.

Daily life problem

Jones, Pevzner: An Introduction to Bioinformatics Algorithms.
MIT Press, 2004
Section 2.9

Exhaustive Search

- You ignore that the phone is ringing
- You walk through every possible angle of the room to find the phone
- You eventually find the phone, but you won't be able to answer

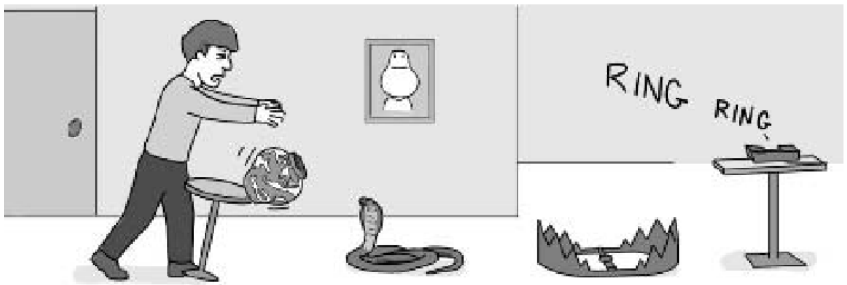
Exhaustive Search

- You ignore that the phone is ringing
- You walk through every possible angle of the room to find the phone
- You eventually find the phone, but you won't be able to answer
- You can optimize such an approach by *omitting or pruning* part the alternatives (e.g., if the phone is ringing above your head, just look everywhere upstairs!)

Exhaustive Search

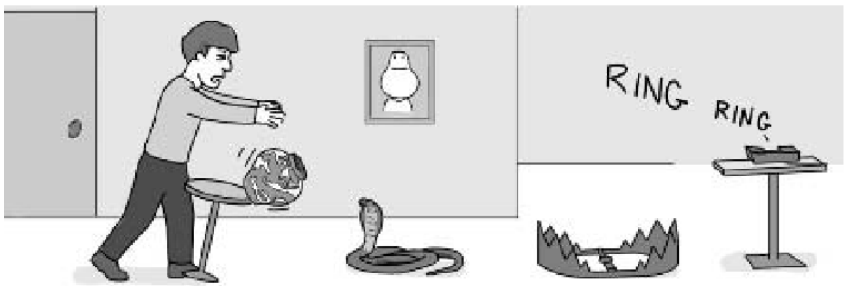
- Is one of the algorithms for sorting doing brute force?
- No, how would that be?

Greedy Algorithms



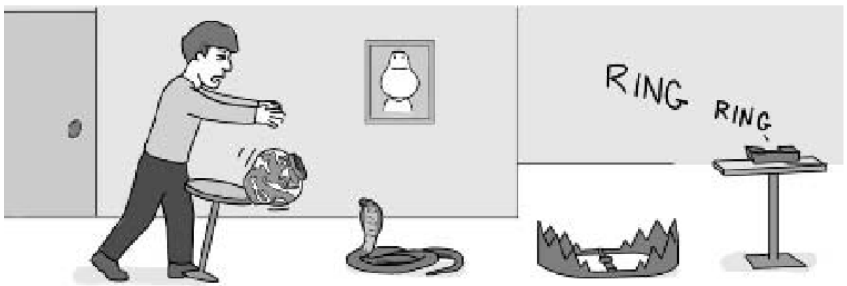
- Walk in the direction of the telephone's ringing until you found it.
- If there is a wall (or an expensive and fragile vase) between you and the phone, prevents you from finding the phone.
- Unfortunately, these sorts of difficulties frequently occur in most realistic problems.

Greedy Algorithms



- Is one of the algorithms for sorting greedy?

Greedy Algorithms



- Is one of the algorithms for sorting greedy?
- Selection sort

Dynamic Programming problems

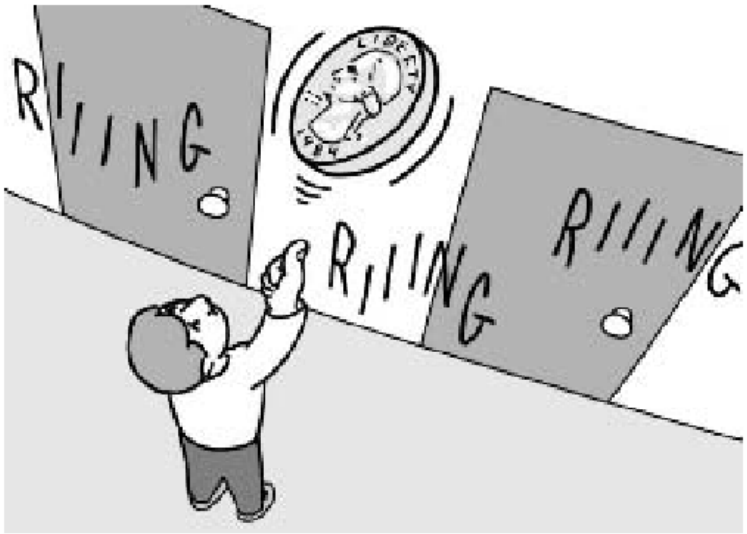
	0	1	2	3	4	5	6	7	8	9	10
0	*	←	*	←	*	←	*	←	*	←	*
1	↑	↖	↑	↖	↑	↖	↑	↖	↑	↖	↑
2	*	←	*	←	*	←	*	←	*	←	*
3	↑	↖	↑	↖	↑	↖	↑	↖	↑	↖	↑
4	*	←	*	←	*	←	*	←	*	←	*
5	↑	↖	↑	↖	↑	↖	↑	↖	↑	↖	↑
6	*	←	*	←	*	←	*	←	*	←	*
7	↑	↖	↑	↖	↑	↖	↑	↖	↑	↖	↑
8	*	←	*	←	*	←	*	←	*	←	*
9	↑	↖	↑	↖	↑	↖	↑	↖	↑	↖	↑
10	*	←	*	←	*	←	*	←	*	←	*

- Split a problem into subproblems and solve each subproblem to solve the general problem (example, insertion sort)
- Not applicable to the problem of the phone.
- You want to turn on to fresh all the rooms of your house.
- You solve the problem room by room by turning on the AC at the proper temperature (depending on sun exposition, for example) in each room.

Divide and conquer Algorithms

- Split the problem of finding the phone into subproblems and solve each subproblem to solve the general problem and then merge the solutions
- We have seen two algorithms working in this way: Merge Sort and Quick Sort
- This approach goes along with recursion

Randomized Algorithms



- Toss a coin to decide whether you want to start your search on the first floor if the coin comes up heads, or on the second floor if the coin comes up tails (you can also use a die).
- We have seen an algorithm working in this way: Quick Sort